PHYSICS NYB-10/11 Winter 2007

Lecture 5: Motion of charged particles in electric fields

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Review of the chapter

- Two types of electric charge: positive and negative
- $|\vec{F}_e| = k_e \dfrac{|q_1||q_2|}{r^2}$ along line joining point charges q_1 and q_2
- Opposites attract, like charges repel
- Conductor: material in which charges are free to move around
- Insulator: material in which charges are not free to move around
- A point charge sets up an electric field $\vec{E} = k_e \dfrac{q}{r^2} \hat{r}$
- $\vec{E}$ is a vector at each point in space, giving us the direction of the force that a positive test charge would feel if placed at that point

Review of the chapter

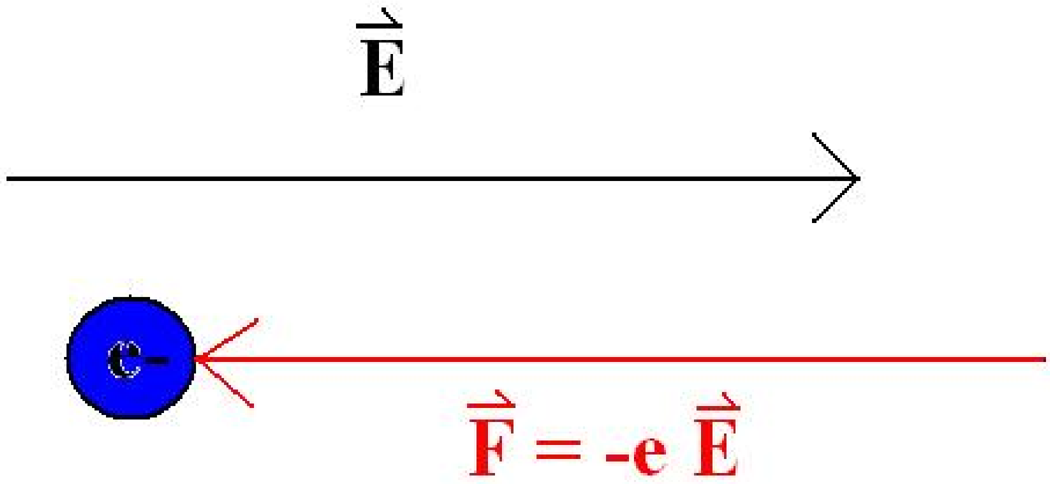
- The force on a charge q_0 placed in a field $\vec{E}$ is $\vec{F}_e = q_0\vec{E}$.
- The force on a charge q_0 from a whole bunch of other charges is the sum of the force from each of the charges $\vec{F}_{tot} = \sum_i \vec{F}_i$
- The field created by a bunch of charges is the sum of the fields from each charge $\vec{E}_{tot} = \sum_i \vec{E}_i$
- The field is a physically real entity that exerts the force, and can carry energy.
- Light is an oscillation in the electromagnetic field.

Review of the chapter

- We can represent the electric field using **field lines**
- At every point in space, the electric field vector is tangential to the field lines
- Field lines begin at positive charges and end on negative charges
- The density of field lines is proportional to the magnitude of $\vec{E}$
- We can use electric fields to accelerate charges, which leads to many useful applications

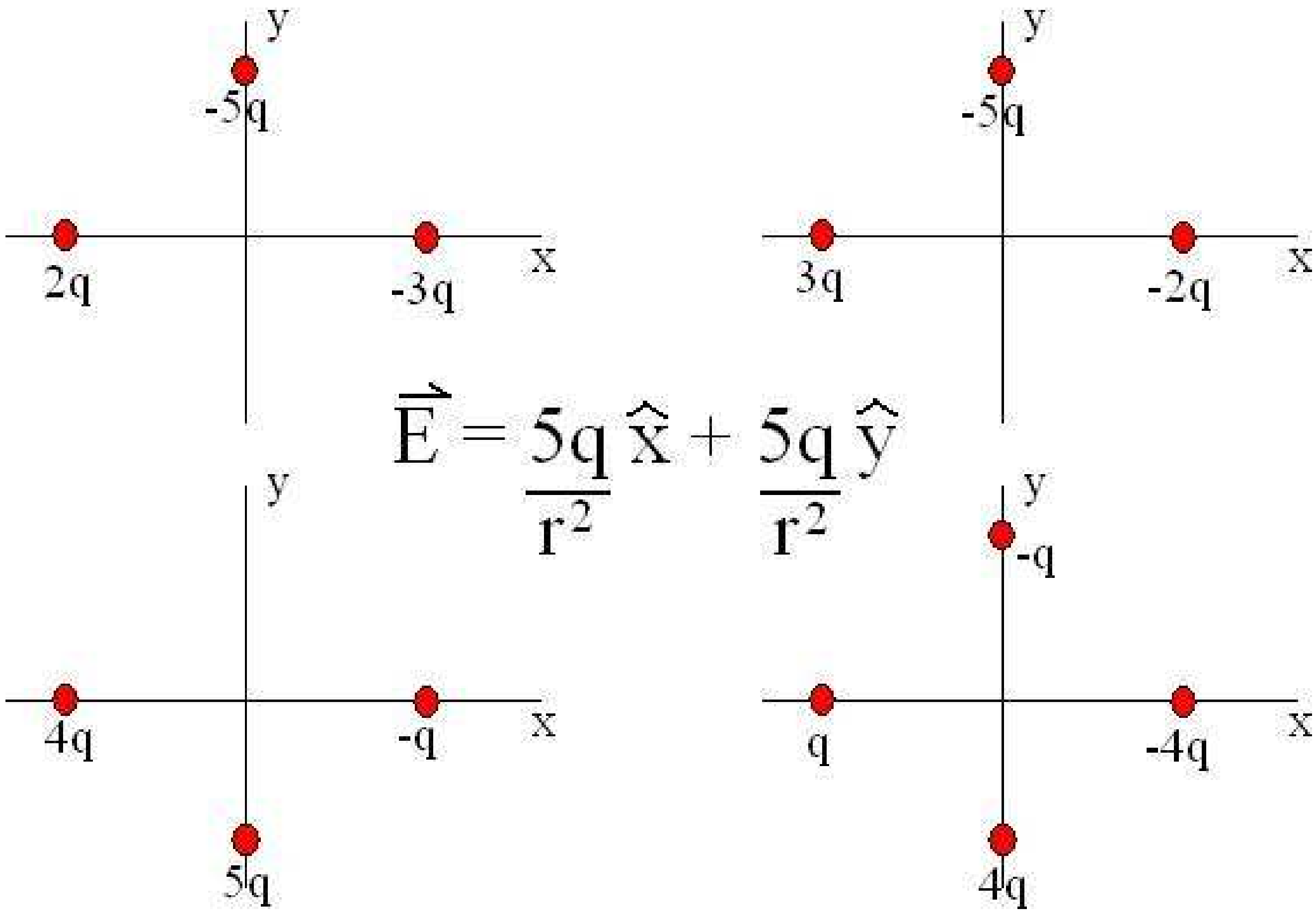
Examples

Question: The figure below shows an electron in a uniform external electric field. What is the direction of the electric force the electron will feel in this field?



Examples

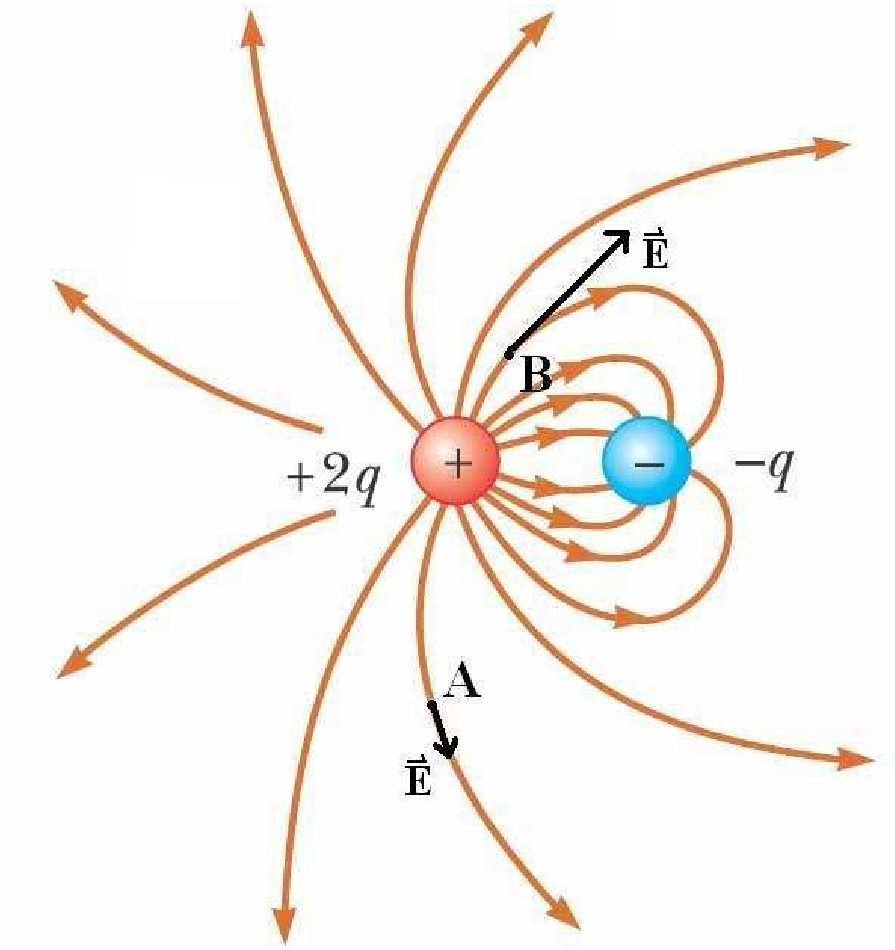
Question: The figure below shows four situations where charged particles are at equal distances from the origin. Ranks the situations according to the magnitude of the net electric field at the origin, greatest first.



$$\vec{E} = \frac{5q}{r^2}\,\hat{x} + \frac{5q}{r^2}\,\hat{y}$$

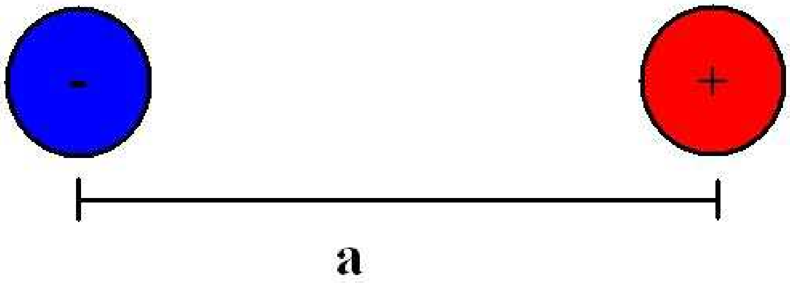
Examples

Question: Draw the electric field vectors at points A and B.



Example

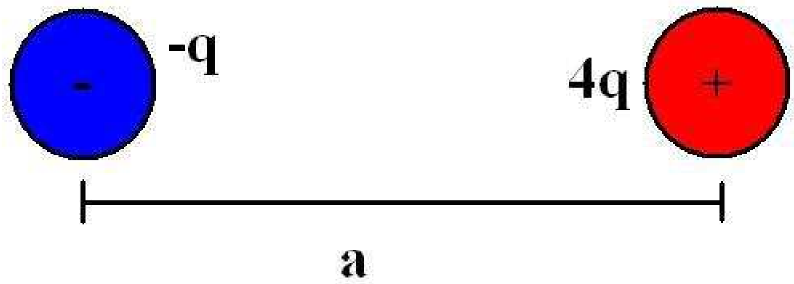
Here, we have pictured a proton and an electron separated by a distance a. Which particle feels the bigger force? Which particle will have the greater acceleration? Is it correct to neglect gravity when answering these two questions?



The force on each is the same! (Remember, this is always the case for objects exerting force *on each other*.) However, the electron has a much smaller mass than the proton, so through $F = ma$, we see that the electron will have greater acceleration than the proton. Gravity between these particles is so weak that it *is* safe to neglect it.

Example

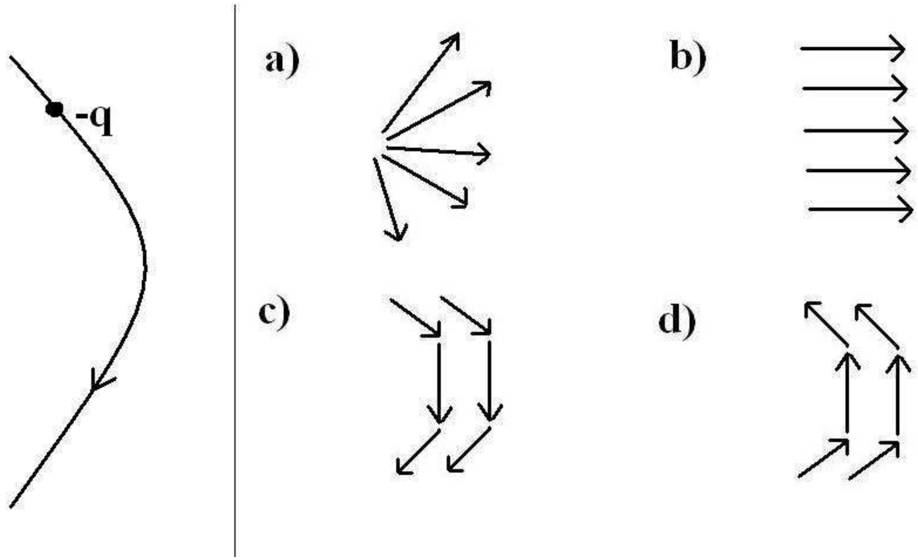
Here, we have pictured two different charges separated by a distance a. Which particle feels the bigger force? Which particle will have the greater acceleration?



The force on each is the same, even if the charges aren't! We can't say anything about the accelerations, since we haven't been told anything about the masses of the particles.

Example

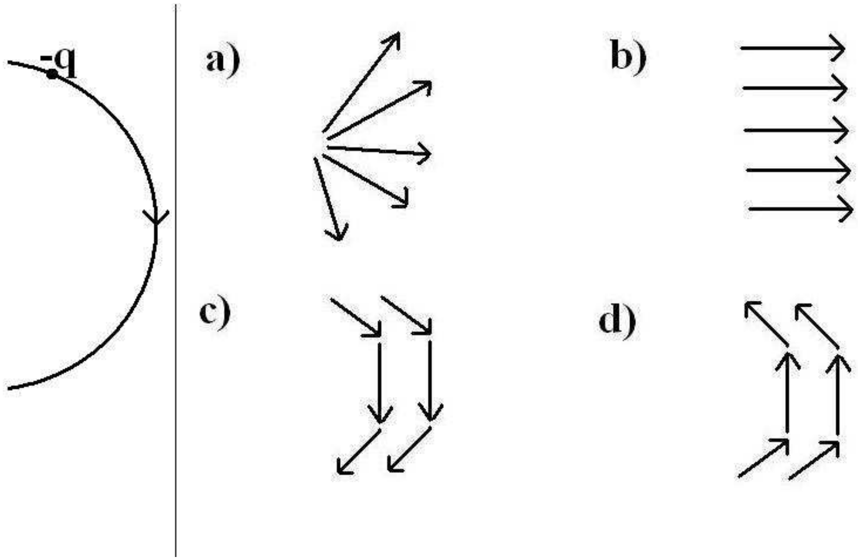
Which of the electric fields pictured below can give rise to parabolic motion of a negative particle, as shown on the left?



You have to remember that the electric field vectors tell us the direction of the *force* felt by charged particles, not their motion. Also, a negative charge feels a force in the direction opposite the electric field vectors. Parabolic motion occurs in a uniform electric field, so the correct choice is b).

Example

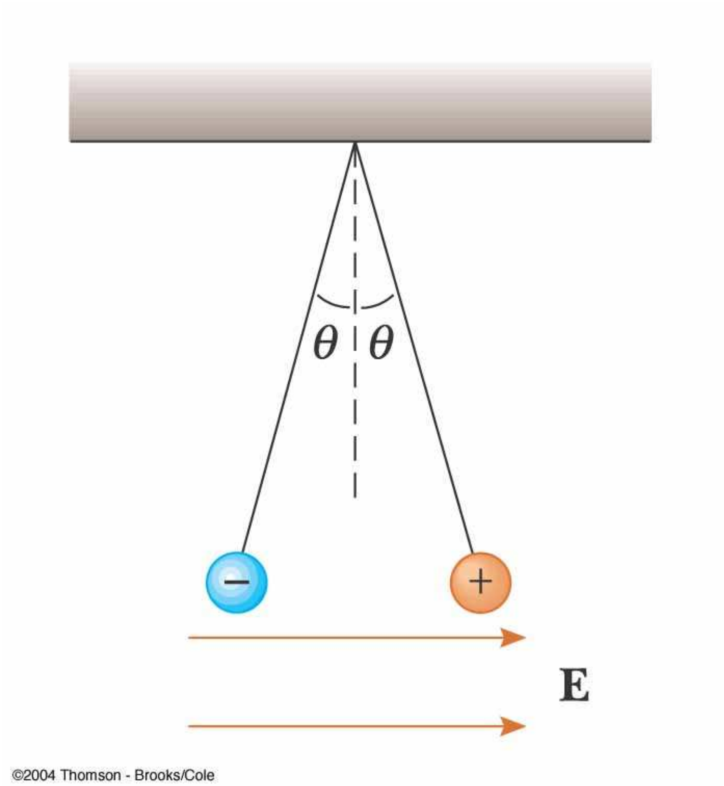
Which of the electric fields pictured below can give rise to circular motion of a negative particle, as shown on the left?



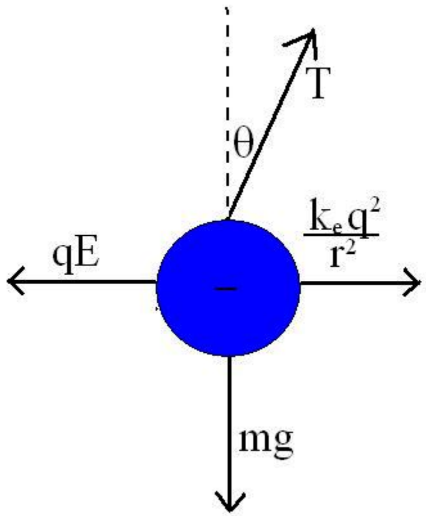
Circular motion requires a force towards a central point, so the correct choice is a).

Example

Two small spheres of mass $m = 2.00$ g are suspended by light strings of length 10.0 cm, as shown below. A uniform electric field is applied in the x-direction. The spheres have charges equal to $\pm 5.00 \times 10^{-8}$ C. What is the electric field if the angle $\theta = 10.0^o$?



Example



x-direction: $q|\vec{E}| = k_e \frac{q^2}{r^2} + T\sin\theta$
y-direction: $mg = T\cos\theta$

$$\Rightarrow T = \frac{mg}{\cos\theta} = 0.020 \text{ N}$$

$$\Rightarrow |\vec{E}| = k_e \frac{q}{(2l\sin\theta)^2} + \frac{mg}{q}\tan\theta = 4.42 \times 10^5 \text{ N/C}$$

Motion of charged particles

A charged particle in an electric field feels a force.

Question: (review of physics NYA!) what happens to a particle of mass m when a net force $\vec{F}$ acts on it?

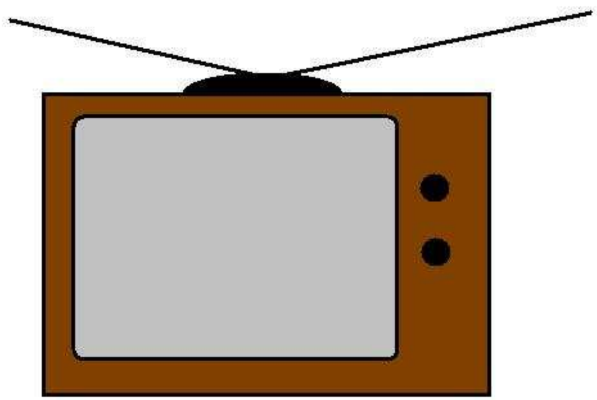
$$\vec{a} = \frac{\vec{F}}{m}$$

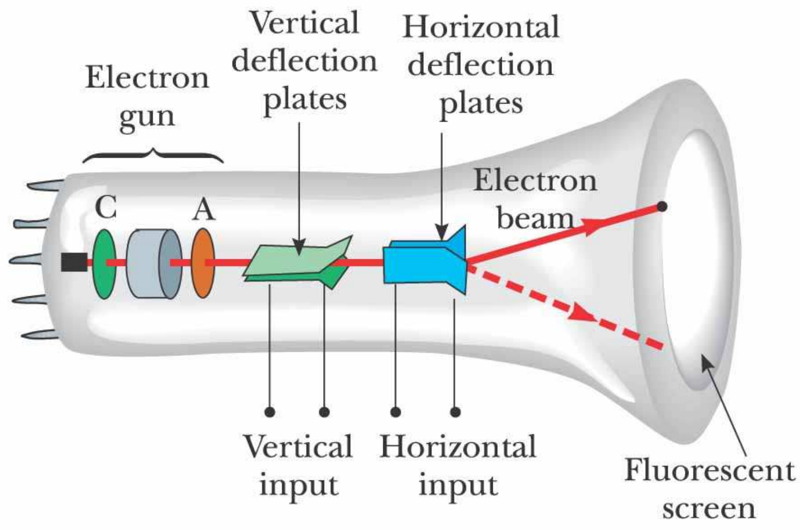
A charged particle's motion through an electric field will therefore be **influenced** by the electric force the particle feels.

Motion of charged particles

There are many interesting applications that rely on putting charged particles inside electric (and magnetic) fields.

<u>Question</u>: Do you know one everyday life object that uses an electric field to accelerate electrons and is found in most homes?





Motion of charged particles

There are also applications to fundamental physics.

The Large Hadron Collider (LHC) in Geneva is the largest particle accelerator in the world and is going to be turned on for the first time in the next few months. It has a circumference of 27 km.

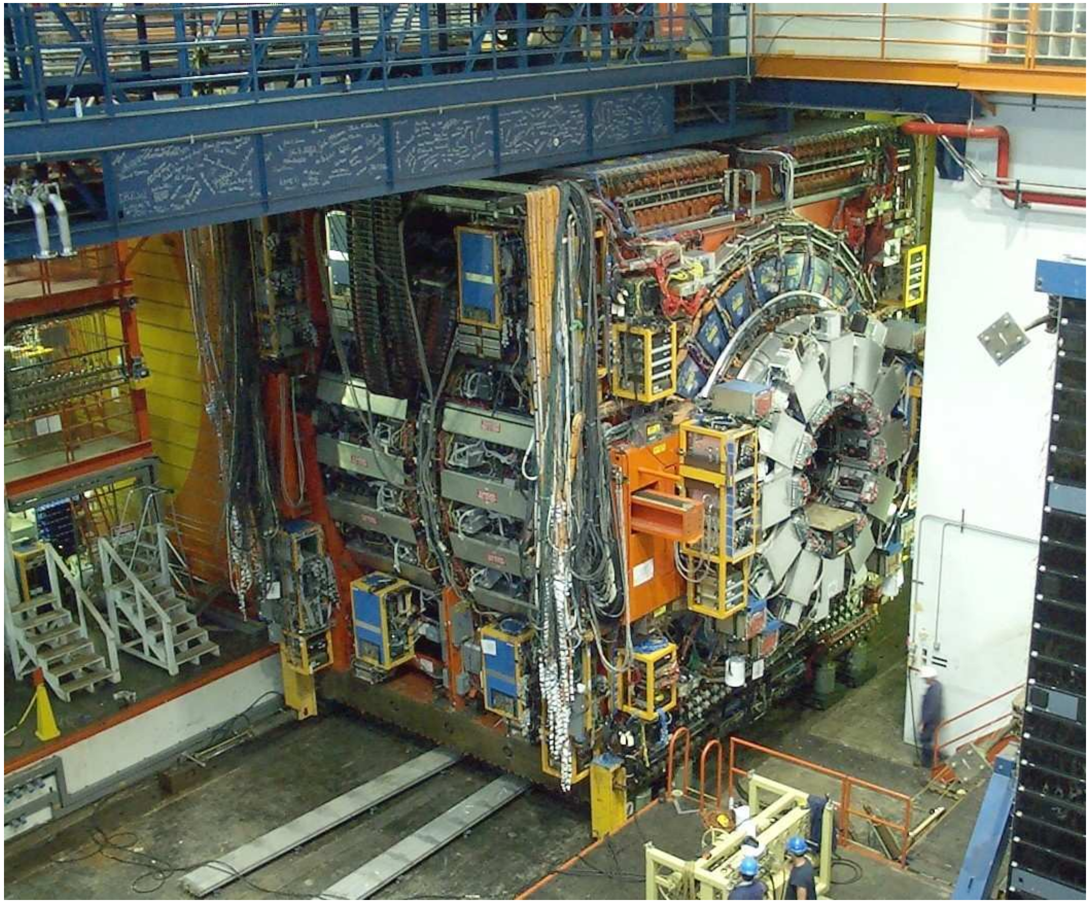
Motion of charged particles

There are also applications to fundamental physics.

It uses electromagnetic fields to accelerate protons to 99.999999% the speed of light.

Motion of charged particles

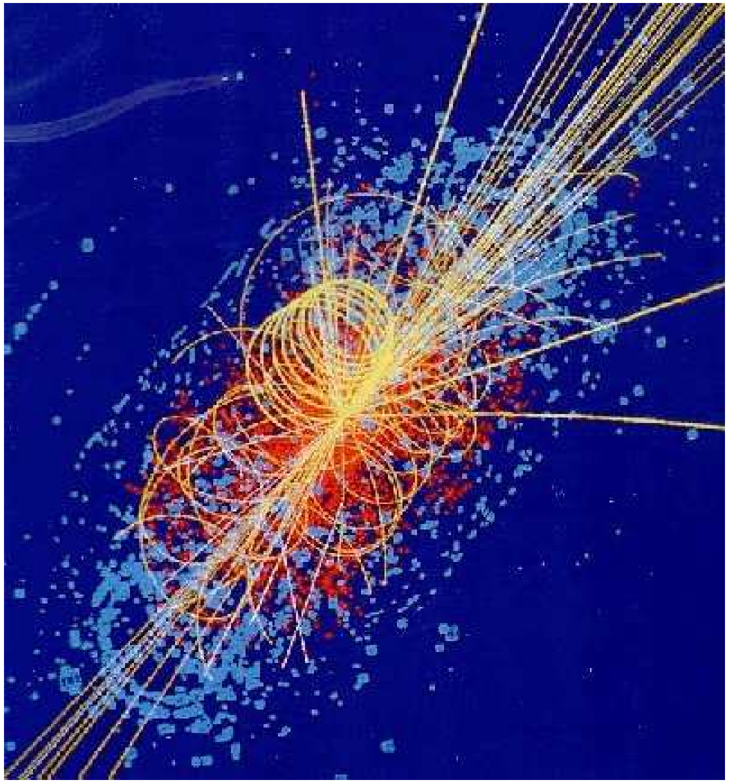
There are also applications to fundamental physics.



The particles smash into each other, and huge detectors will keep track of what comes out.

Motion of charged particles

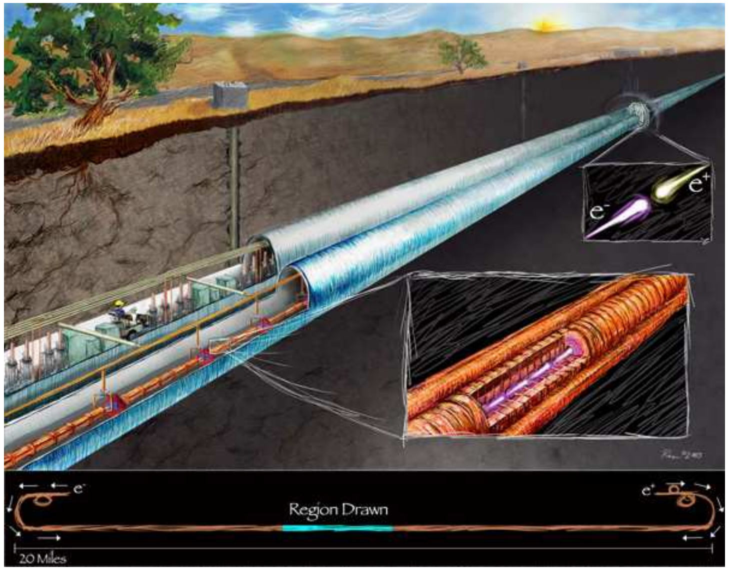
There are also applications to fundamental physics.



These collisions will recreate conditions that existed in the Universe about 10^{-11} seconds after the Big Bang!

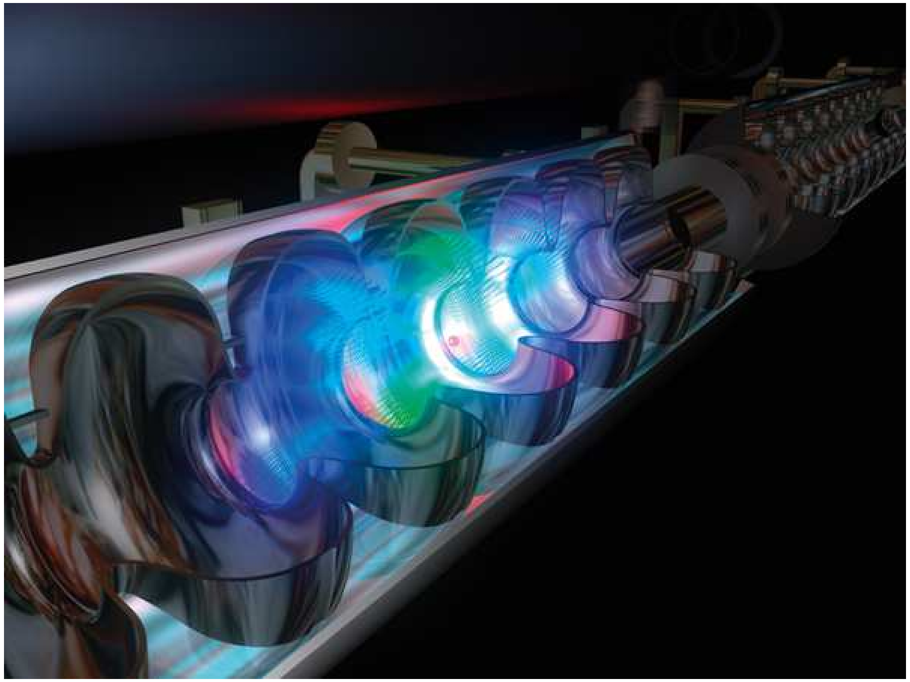
Motion of charged particles: example

As impressive as the LHC is, physicists are already planning the next generation particle accelerator, the ILC (International Linear Collider). A linear collider has the advantage of not having the particles lose energy because of their curved trajectory. However, the particles can't be accelerated repeatedly by going around the same circle a number of times, so a very long collider and strong fields are required.



Motion of charged particles: example

Assuming the electric field in the ILC was a simple uniform electric field, and neglecting relativistic effects (which we can't do!!!), what magnitude would be required for the planned 35 km long ILC to accelerate electrons and positrons to $0.999\,999\,999\,999\,5$ times the speed of light?

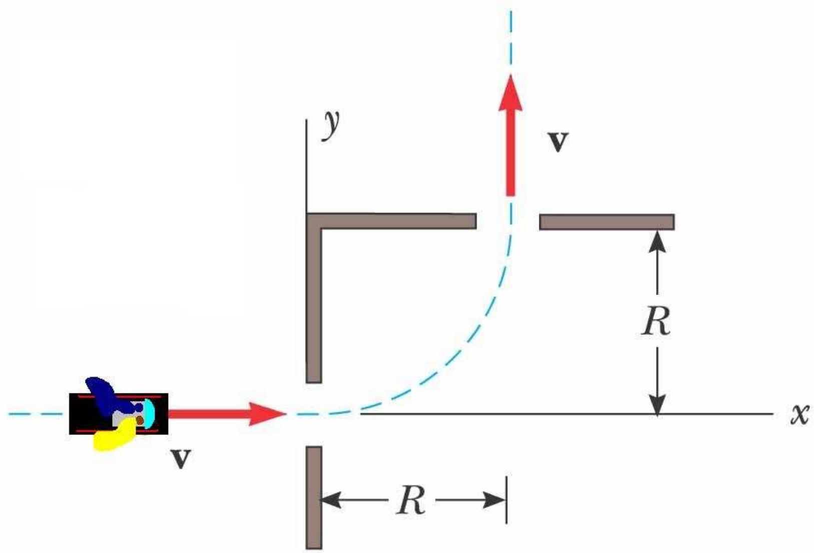


Motion of charged particles: example

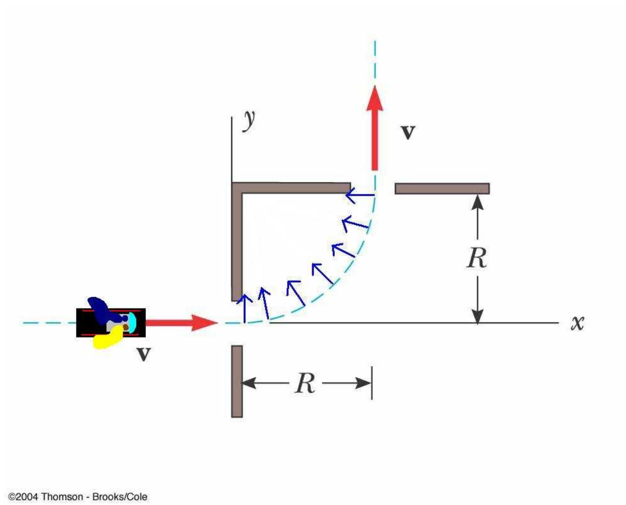
First, since the particles collide in the middle, they actually must be accelerated over 17.5 km, not 35 km. Remembering from physics NYA that $v_f^2 = v_i^2 + 2a\Delta x$, we find that $a = \frac{v_f^2}{2\Delta x} = 2.57 \times 10^{12}$ m/s^2. This requires a force of $F = ma = 9 \times 10^{-31}$ kg $\times\, 2.57 \times 10^{12}$ m/s$^2 = 2.34 \times 10^{-18}$ N. If this is to come from the electric force, $|\vec{F}| = q|\vec{E}|$, we get $|\vec{E}| = \frac{2.34 \times 10^{-18}}{1.6 \times 10^{-19}} = 14.6$ N/C. This is actually not a large electric field. However, we neglected relativity. Taking relativity into account, the answer is closer to 28.5×10^6 N/C!!!

Motion of charged particles: example

Batman and Robin are racing into the Batcave after an ice-storm. There is no friction between the bat-wheels and the bat-floor, yet they must round a curve as shown below. Luckily, the Batmobile picked up a charge q from friction with the air during their drive. They will use an electric field to make the Batmobile round the curve. Which of a radial and a uniform electric field will need a smaller magnitude? (Treat the Batmobile as a point charge, of course).



Motion of charged particles: example



If the field is radial, and the Batmobile undergoes circular motion. Remembering NYA we know that this means there is a force

$$\vec{F} = -\frac{m_B v^2}{R}\hat{r}$$

acting on it. This means that the electric field must be

$$\vec{E} = \frac{\vec{F}}{q} = -\frac{m_B v^2}{qR}\hat{r}$$

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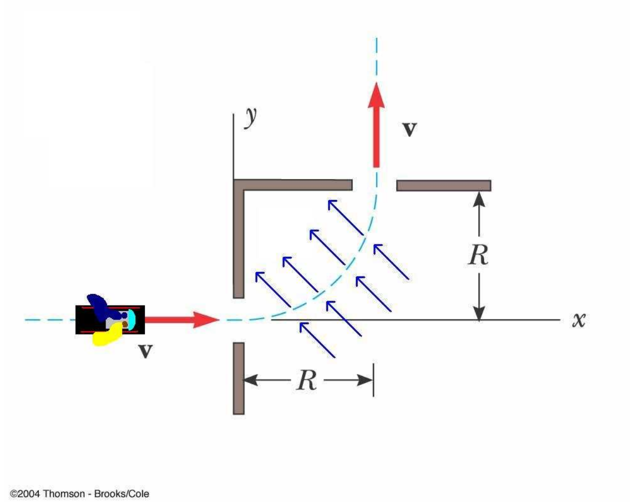
Motion of charged particles: example



If the field is constant, we want v_x to diminish to from v to 0 over a distance R, and we want v_y to increase from 0 to v over a distance R. Since $v_f^2 = v_i^2 + 2a(\Delta x)$,

$$\begin{aligned} 0 &= v^2 + 2a_x R \Rightarrow a_x = \frac{F_x}{m_B} = \frac{qE_x}{m_B} = -\frac{v^2}{2R} \\ v^2 &= 0 + 2a_y R \Rightarrow a_y = \frac{F_y}{m_B} = \frac{qE_y}{m_B} = \frac{v^2}{2R} \end{aligned}$$

Motion of charged particles: example

So in this second case, $\vec{E} = -\frac{m_B v^2}{2qR}\hat{i} + \frac{m_B v^2}{2qR}\hat{j}$, and the magnitude is $\sqrt{2 \times \left(\frac{m_B v^2}{2qR}\right)^2} = \frac{m_B v^2}{\sqrt{2}qR}$, while in the first case the magnitude is $\frac{m_B v^2}{qR}$. A uniform field therefore requires a lesser magnitude.

Assignment 2

- Chapter 23, problems 19, 21, 22, 40, 48, 51, 56

What to read for next lecture

- 25.1, 25.2